Schutz 8.5 (a).

$$R = \frac{1}{2}\left(h_{\alpha\nu,\beta\mu} + h_{\beta\mu,\alpha\nu} - h_{\alpha\mu,\beta\nu} - h_{\beta\nu,\alpha\mu}\right).$$

$$h_{\alpha\beta} = \xi_{\alpha,\beta} + \xi_{\beta,\alpha}.$$

$$\Rightarrow R \sim \left[\begin{array}{l} \xi_{\alpha,\nu\beta\mu} + \xi_{\nu,\alpha\beta\mu} + \xi_{\beta,\mu\alpha\nu} + \xi_{\mu,\beta\alpha\nu} \\ -\xi_{\alpha,\mu\beta\nu} - \xi_{\mu,\alpha\beta\nu} - \xi_{\beta,\nu\alpha\mu} - \xi_{\nu,\beta\alpha\mu} \end{array}\right] = \boxed{0}.$$

$\Uparrow$

The corresponding canceling terms are indicated by the arrows.

(b) According to (8.24), under gauge trans, $h_{\alpha\beta} \to h_{\alpha\beta} - (\xi_{\alpha,\beta} + \xi_{\beta,\alpha})$

$$\Rightarrow \delta R \sim -\left(\sigma_{\alpha\nu,\beta\mu} + \sigma_{\beta\mu,\alpha\nu} - \sigma_{\alpha\mu,\beta\nu} - \sigma_{\beta\nu,\alpha\mu}\right),$$

where $\sigma_{\alpha\nu} = \xi_{\alpha,\nu} + \xi_{\nu,\alpha}$

$\Rightarrow \boxed{\delta R = 0.}$ according to part (a).

Note we introduced σ which is equivalent to h in part (a) to avoid confusion.

Davidson Cheng

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